GR 1

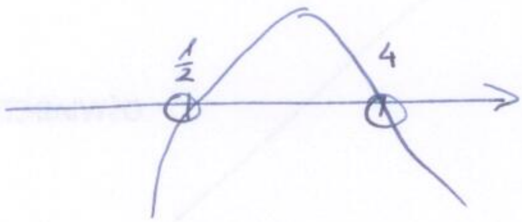
(1)

$$\frac{2x-1}{4-x} > 0 \quad \wedge \quad \underline{x \neq 4}$$

$$(2x-1)(4-x) > 0$$

$$-2\left(x-\tfrac{1}{2}\right)(x-4) > 0$$

$x = \frac{1}{2}$ $\qquad x = 4$



$$\underline{\underline{x \in \left(\tfrac{1}{2}; 4\right)}}$$

(2)

$y = \frac{1}{x}$ $\qquad D_y = \mathbb{R} \setminus \{0\}$

$\qquad \qquad \mathbb{II}_y = \mathbb{R} \setminus \{0\}$

$D_{y^{-1}} = \mathbb{R} \setminus \{0\}$

$\mathbb{II}_{y^{-1}} = \mathbb{R} \setminus \{0\}$

$\underline{x = \frac{1}{y}} = y^{-1}$

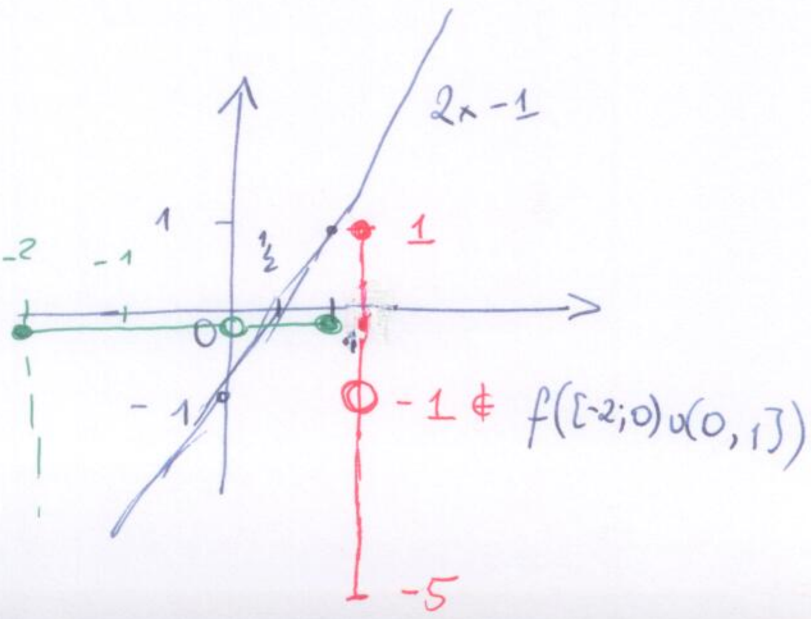
(3)

$f(x) = 2x - 1$

$\cancel{f(2x)}$

$f([-2;0) \cup (0,1]) =$

$= [-5;-1) \cup (-1;1]$



(5a)

GR 1

$\ln(1+x) - \ln(1-x) = 1$

$\mathbb{D}:$ $\begin{cases} 1+x>0 \\ 1-x>0 \end{cases} \Rightarrow \begin{matrix} x>-1 \\ x<1 \end{matrix}$ $\mathbb{D} = (-1; 1)$

$\ln\left(\frac{1+x}{1-x}\right) = \ln e$

$\frac{1+x}{1-x} - e = 0$

$\frac{1+x-e(1-x)}{1-x} = 0$

$\frac{1+x-e+ex}{1-x} = 0$

$\frac{1-e+(1+e)x}{1-x} = 0 \iff 1-e+(1+e)x = 0$

$x = -\frac{1-e}{1+e} = \frac{e-1}{1+e}$

$x \in \mathbb{D}$

(5b)

$6^{2x+4} = 2^{x+8}\ 3^{3x}$

$6^4 \cdot 3^{2x} \cdot 2^{2x} = 2^8\ 2^x \cdot 3^{3x} \quad \Big| : 3^{2x} \cdot 2^{2x}$

$6^4 = 2^8\ \frac{2^x \cdot 3^{3x}}{3^{2x} \cdot 2^{2x}}$

$6^4 = 2^8 \cdot \frac{3^x}{2^x}$

$\left(\frac{3}{2}\right)^x = \frac{3^4 \cdot 2^4}{2^8} = \left(\frac{3}{2}\right)^4$

$x = 4$

5c

$$2^{2x-4} - 5\,2^{x-1} + 16 = 0$$

GR 1

$$2^{-4}\cdot(2^x)^2 - 5\cdot\tfrac{1}{2}\cdot 2^x + 16 = 0$$

$$t = 2^x, \quad t > 0$$

$$2^{-4}t^2 - 5\cdot\tfrac{1}{2}\cdot t + 16 = 0$$

$$\Delta = \frac{25}{4} - 4\cdot 16\cdot\frac{1}{16} = \frac{25}{4} - \frac{16}{4} = \frac{9}{4}$$

$$t_1 = \frac{+\frac{5}{2} - \frac{3}{2}}{2^{-3}} = 2^3$$

$$t_2 = \frac{\frac{5}{2} + \frac{3}{2}}{2^{-3}} = 2^5$$

$$2^{x_1} = 2^3 \quad \vee \quad 2^{x_2} = 2^5$$

$$x = 3 \quad \vee \quad x = 5$$

6a

$$2^{2x+1} - 3\cdot 2^{x+1} + 4 \leq 0$$

$$2\cdot(2^x)^2 - 3\cdot 2\cdot 2^x + 4 \leq 0$$

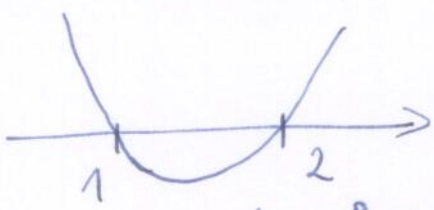
$$t = 2^x, \quad t > 0$$

$$2\cdot t^2 - 6t + 4 \leq 0$$

$$\Delta = 36 - 4\cdot 8 = 4$$

$$t_1 = \frac{6-2}{4} = 1$$

$$t_2 = \frac{6+2}{4} = 2$$



$$t \in [1, 2]$$

$$2^x = 1 \Rightarrow 2^x = 2^0$$

$$2^x = 2 \Rightarrow 2^x = 2^1$$

$$x \in (0, 1)$$

(6b)

$\log_2(x-5) + \log_2(2-x) < \cancel{1}$ $\boxed{GR1}$

$D: \begin{cases} x-5>0 \\ 2-x>0 \end{cases} \Rightarrow \begin{cases} x>5 \\ x<2 \end{cases}$ brak rozwiązań

(6c)

$\log_{1/3}(4x+1) > -2 - \log_{1/3}(2x-3)$

$D:$

$\begin{cases} 4x+1>0 \\ 2x-3>0 \end{cases} \quad \begin{cases} x>-\frac{1}{4} \\ x>\frac{3}{2} \end{cases} \Rightarrow D=\left(\frac{3}{2};+\infty\right)$

$\log_{1/3}(4x+1) > -\log_{1/3}\left(\frac{1}{3}\right)^2 - \log_{1/3}(2x-3)$

$\cancel{\log_{1/3}(4x+1) + \log_{1/3}\left(\left(\frac{1}{3}\right)^2(2x-3)\right) >}$

$\log_{1/3}(4x+1) > -\left[\log_{1/3}\left(\frac{2x-3}{9}\right)\right]$

$\log_{1/3}(4x+1) > \log_{1/3}\left(\frac{9}{2x-3}\right)$

$\cancel{\log_{1/3}}\ 4x+1 < \frac{9}{2x-3}$

$\frac{(4x+1)(2x-3)-9}{2x-3} < 0$

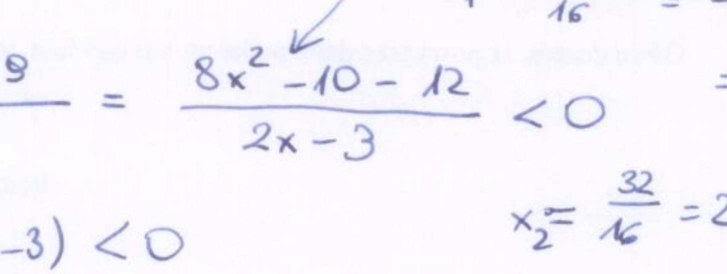
$\Delta = 100 + 4\cdot 8\cdot 12 = 484$

$x_1 = \frac{10-22}{16} = -\frac{12}{16} = -\frac{6}{8} = -\frac{3}{4}$

$\frac{8x^2-12x+2x-3-9}{2x-3} = \frac{8x^2-10-12}{2x-3} < 0$

$x_2 = \frac{32}{16} = 2$

$8\left(x+\frac{3}{4}\right)(x-2)(2x-3) < 0$



$x \in \left(-\infty; -\frac{3}{4}\right) \cup \left(\frac{3}{2}; 2\right)$

Odp. $x \in \left(\frac{3}{2}; 2\right)$

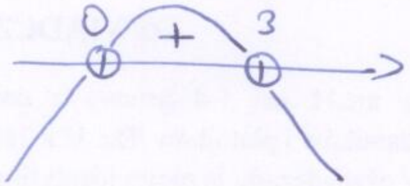
① GR2

$$f(x) = \log\left(\frac{x}{3-x}\right)$$

$$D: \begin{cases} \frac{x}{3-x} > 0 \\ x \neq 3 \end{cases}$$

$$-x(x-3) > 0$$

$$x \in (0,3)$$



②.

$$f(x) = \frac{2}{x} \qquad f^{-1}(y) = \frac{2}{y}, \text{ bo}$$

$$D_f = \mathbb{R}\setminus\{0\} = W_{f^{-1}} \qquad y = \frac{2}{x} \quad / \cdot x$$

$$W_f = \mathbb{R}\setminus\{0\} = D_{f^{-1}} \qquad xy = 2 \quad /: y \quad , y \neq 0$$

$$x = \frac{2}{y}$$

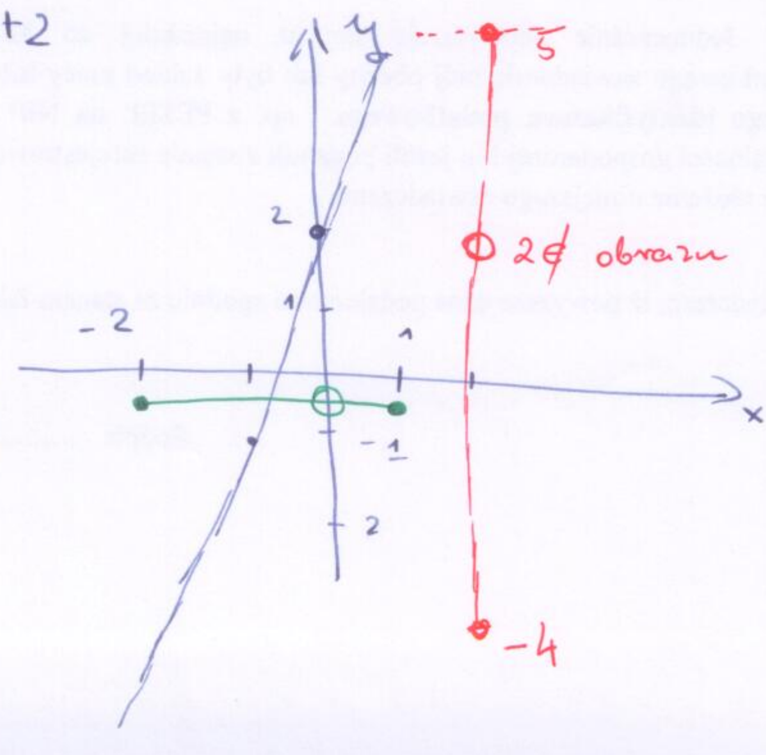
③.

$$f(x) = 3x+2$$

~~fog~~

$$f([-2;0)\cup(0;1])$$

$$= [-4;2)\cup(2;5]$$



(5a) $\log(4+x) - 2\log x = 0$ GR2

D $\begin{cases} 4+x>0 \\ x>0 \end{cases} \Rightarrow \begin{cases} x>-4 \\ x>0 \end{cases}$ $\underline{x>0}$

$\log(4+x) = \log x^2$

$4+x-x^2=0$

$x^2-x-4=0$ $\Delta = 1+16=17$

$x_1 = \frac{1-\sqrt{17}}{2} \notin D$

$x_2 = \frac{1+\sqrt{17}}{2}$

Rozw. $x = \frac{1+\sqrt{17}}{2}$

(5b)

$10^{2x+4} = 2^{x+8} \cdot 5^{3x}$

$10^4 \cdot 5^{2x} \cdot 2^{2x} = 2^8 \cdot 2^x \cdot 5^{3x}$ $/ : 2^{2x} \cdot 5^{2x} \cdot 2^8$

$\frac{5^4 \cdot 2^4}{2^8} = \frac{2^x \cdot 5^{3x}}{2^{2x} \cdot 5}$

$\left(\frac{5}{2}\right)^4 = \left(\frac{5}{2}\right)^x$

(5c) GR2

$2^{2x+1} - 3\cancel{x}\cdot 2^{x-1} - 14 = 0$

$2\cdot(2^{x})^{2} - 3\cdot 2^{-1}\cdot 2^{x} - 14 = 0$

$t = 2^{x}, \quad t > 0$

$2\cdot t^{2} - \frac{3}{2}t - 14 = 0 \quad /:2$

$\cancel{\Delta = \frac{9}{4} + 4\cdot 2\cdot 14 =}$

$t^{2} - \frac{3}{4}t - 7 = 0$

$\Delta = \frac{9}{16} + 4\cdot 7 = \frac{457}{16} = \cancel{\frac{1}{4}\sqrt{\ }}$

$t_1 = \frac{\frac{3}{4} - \frac{1}{4}\sqrt{457}}{2} = \frac{3-\sqrt{457}}{8} < 0$

$t_2 = \frac{3+\sqrt{457}}{8} > 0$

$t = \frac{3+\sqrt{457}}{8}$

$2^{x} = \frac{3+\sqrt{457}}{8}$

$x = \cancel{\ln} \log_2\left(\frac{3+\sqrt{457}}{8}\right)$

(6a)

$3^{2x} - \frac{1}{3}\cdot 3^{x} - \frac{2}{3} > 0$

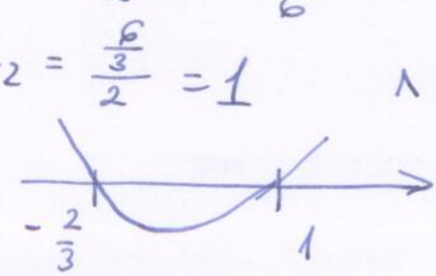
$t = 3^{x}, \quad t > 0$

$t^{2} - \frac{1}{3}\cdot t - \frac{2}{3} > 0$

$\Delta = \frac{1}{9} + \frac{8}{3} = \frac{1}{9} + \frac{24}{9} = \frac{25}{9}$

$t_1 = \frac{\frac{1}{3} - \frac{5}{3}}{2} = -\frac{4}{6}$

$t_2 = \frac{\frac{6}{3}}{2} = 1 \quad \wedge \ t > 0$



$t > 1$

$3^{x} > 1$

$3^{x} > 3^{0}$

$\underline{x > 0}$

6b

GR2

$$\log_3 (2x-7) \le 2 - \log_3 (8-x)$$

$$D: \begin{cases} 2x-7>0 \\ 8-x>0 \end{cases} \Rightarrow \begin{cases} x > \frac{7}{2} \\ x < -8 \end{cases}$$

$$x \in \emptyset$$

6c

$$\log_{\frac{1}{2}} (2x-6) - 2\log_{\frac{1}{2}} (x-3) \ge 0$$

$$\log_{\frac{1}{2}} (2x-6) \ge 2\log_{\frac{1}{2}} (x-3)$$

$$D: \begin{cases} 2x-6>0 \\ x-3>0 \end{cases} \Rightarrow \begin{cases} x>3 \\ x>3 \end{cases} \Rightarrow D = (3; +\infty)$$

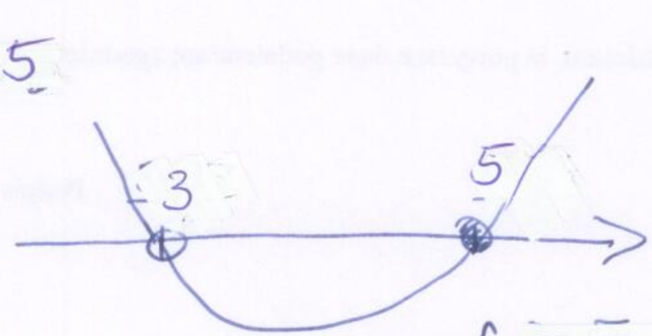
$$\log_{\frac{1}{2}} (2x-6) \ge \log_{\frac{1}{2}} (x-3)^2$$

$$2x-6 \le (x-3)^2 = x^2 - 6x + 9$$

$$x^2 - 8x + 15 \ge 0 \qquad \Delta = 64 - 60 = 4$$

$$x_1 = \frac{8-2}{2} = 3$$

$$x_2 = \frac{8+2}{2} = 5$$



$$x \in [5; +\infty)$$